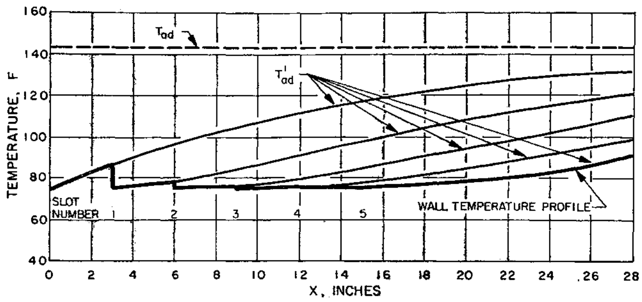

Fig. 6 Calculated temperature profiles for run where n = 5 and s = 3 in.

success in correlating gaseous film-cooling data obtained under full-scale rocket engine conditions.

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Similar and Nonsimilar Solutions of the Nonequilibrium Laminar Boundary Layer

F. G. Blottner*

General Electric Company, Philadelphia, Pa.

Nomenclature

c_i = mass fraction of species i
c_{pi} = specific heat of species i, ft²/(sec²-°R)
$\bar{c}_p$ = specific heat of the mixture, ft²/(sec²-°R)
f = stream function variable defining velocity, $f = \int(u/u_e)d\eta$
h_i = static enthalpy of species i, (ft/sec)²
Le = Lewis-Semenov number
p = static pressure, psf
Pr = Prandtl number
R = universal gas constant, psf/(lb-mole-sec²-°R)
T = absolute temperature, °R
u = component of velocity parallel to surface, fps
$\dot{w}_A$ = net rate of production of atoms, slug/ft³/sec
x = distance along body surface, ft
β = pressure gradient parameter, $(2x/u_e)(du_e/dx)$
θ = temperature ratio, T/T_e
ρ = gas density, slug/ft³
η = nondimensional coordinate normal to body surface, $\eta = [u_e/2x(\rho\mu)_r]^{1/2}\int\rho dy$

Subscripts

A = atoms
e = at the edge of the boundary layer
eq = at local equilibrium
M = molecules
w = at the wall

A prime denotes differentiation with respect to η.

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* Research Engineer, Missile and Space Division, Space Sciences Laboratory. Member AIAA.

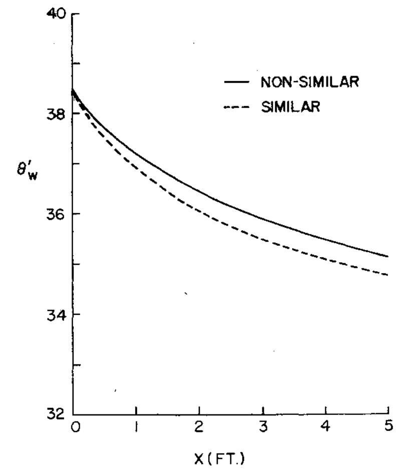

Fig. 1 Temperature gradient at the wall.

THE results of nonsimilar laminar boundary-layer flow with finite chemical reactions along a wall with zero pressure gradient are reported. In addition, the validity of employing the concept of local similarity for chemical nonequilibrium flow is examined by a direct comparison of the nonsimilar solution with a locally similar one.

The results are for a binary mixture (oxygen is this note) with concentration diffusion, a constant Lewis-Semenov number of 1.4, and a constant Prandtl number of 0.7. In the numerical results the specific heat of the mixture, $\bar{c}_p = c_A(c_{pA} - c_{pM}) + c_{pM}$, is taken as a constant with the same value as the specific heat of the atoms, c_{pA}, and the molecules, c_{pM}. With the nondimensional density-viscosity product equal to one, the momentum, energy, and conservation of atoms equations for local similarity become

$$f''' + ff'' = 0$$

$$\frac{\theta''}{Pr} + f\theta' + \frac{u_e^2}{\bar{c}_p T_e}(f'')^2 - \frac{2x}{u_e}\left(\frac{\dot{w}_A}{\rho}\right)\frac{(h_A - h_M)}{\bar{c}_p T_e} = 0$$

$$\frac{Le}{Pr} C_A'' + fC_A' + \frac{2x}{u_e}\left(\frac{\dot{w}_A}{\rho}\right) = 0$$

where

$$\frac{\dot{w}_A}{\rho} = -\frac{3.39 \times 10^{22}}{(T_e\theta)^4}\left(\frac{p_e}{R}\right)^2 \times \left[\frac{C_A^2}{1 + C_A} - 2116.216\,(1 - C_A)\exp\left(15.8 - \frac{108{,}000}{T}\right)\right]$$

The boundary conditions at a catalytic wall with specified temperature are $f(0) = 0$, $f'(0) = 0$, $\theta(0) = \theta_w$, $C_A(0) = C_{A_{eq}}(\theta_w, p_e)$ and at the outer edge are $f'(\eta_e) = 1$, $\theta(\eta_e) = 1$, and $C_A(\eta_e) = C_{A_e}$.

Locally similar solutions are obtained by solving the forementioned set of ordinary differential equations with two-

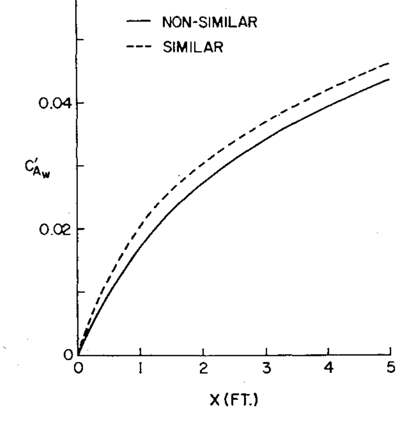

Fig. 2 Atom mass fraction gradient at the wall.

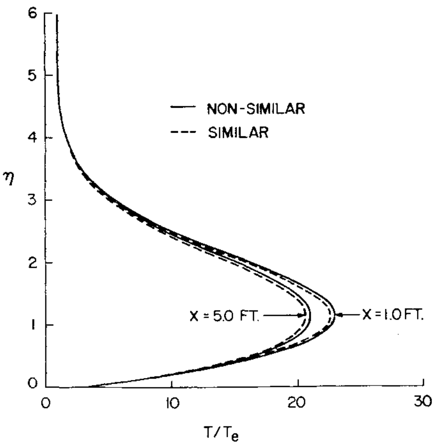

Fig. 3 Temperature profiles at x = 1 ft and 5 ft in transformed coordinate.

point boundary conditions at various distances along the wall. These solutions have been obtained by the usual method of assuming the boundary conditions at the wall and iterating until the boundary conditions at the outer edge are satisfied. In this way, locally similar solutions have been obtained to $x = 5$ ft. However, even at this distance, the time and effort required to obtain the correct iterated boundary conditions is enormous and increases downstream from the leading edge since exceedingly accurate assumed boundary conditions are required there. To obtain these quantities the similar boundary-layer equations are solved near the leading edge where less accurate boundary conditions are required. Then one proceeds along the wall from the frozen toward the equilibrium regime extrapolating the boundary conditions and obtaining solutions further downstream.

To solve the complete partial differential equations of the nonequilibrium boundary layer an implicit finite difference procedure has been developed[1] which accounts for the "nonsimilar" terms not included in the forementioned system of equations. The flow along a wall with zero pressure gradient at an altitude of 100,000 ft (T_e = 392.4°R and p_e = 23.4265 psf) and a freestream velocity of 25,000 fps has been obtained with this numerical scheme. As these results employ the same assumptions about the gas as the similar solutions and approach the exact solution to the problem, the validity of using local similarity for this problem is evaluated. In Figs. 1 and 2 the temperature and atom mass fraction gradients at the wall are presented. These are the boundary conditions at the wall for the similar solution that must be assumed and iterated until the correct values are obtained. For this problem, the velocity profiles are obviously the Blasius result and do not change along the wall. The temperature and atom mass fraction profiles at x = 1 ft and 5 ft are presented in Figs. 3 and 4. These results indicate that the atom mass fraction profiles for the boundary layer with the local similarity assumption are in greatest error.

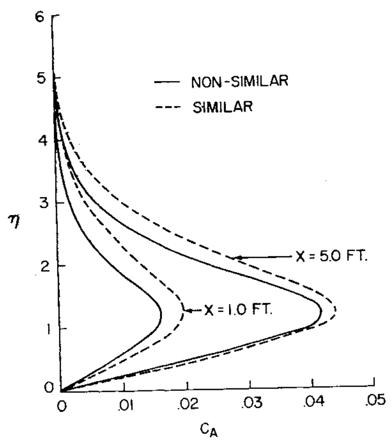

Fig. 4 Atom mass fraction profiles at x = 1 ft and 5 ft in transformed coordinate.

Even if one is willing to accept the accuracy of the similar solution, which is reasonable for this example, the difficulty of acquiring the results limits the usefulness of the procedure. The nonsimilar procedure gives more accurate results while requiring less time to obtain them.

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Effect of Gravity on the Mobility of a Lunar Vehicle

George A. Costello*
University of Illinois, Urbana, Ill.
AND
Donald L. Dewhirst†
Chrysler Corporation, Detroit, Mich.

This paper presents a dimensional analysis for determining the effect of gravity on the tractive effort of a vehicle moving in a soil. The theory shows that the effect of lunar gravity may be simulated by changing controlled variables. Test results are presented for a six-wheel model operating in ¼-in. gravel. These results show that, for a vehicle traveling in a horizontal plane on a cohesionless soil, the percent slip, for a given ratio of drawbar pull to vehicle weight, is practically unaffected by changes in gravity.

Introduction

Simulation of lunar g can be accomplished by means of an accelerating platform, that is, by dropping the test apparatus at $\frac{5}{6}$ g. However, this simulation must be necessarily of short duration.

In this paper, the effects of gravity are determined by employing dimensional analysis and test results. Dimensionless parameters are introduced and the effect of gravity is determined by varying these parameters.

Analysis

The variables that are assumed to be pertinent to the problem under consideration are listed below. The dimensions are given in terms of mass M, length L, and time T.

V = vehicle velocity, LT^{-1}
DP = drawbar pull, MLT^{-2}
M = mass of vehicle, M
g = acceleration due to gravity, LT^{-2}
μ = coefficient of friction, soil to wheel material (dimensionless)
R = radius of wheel, L
ω = angular velocity of the wheel, T^{-1}
% slip = $(R\omega - V)/R\omega$ (dimensionless)
r = soil particle size, L
c = cohesion, $ML^{-1}T^{-2}$
ϕ = angle of internal friction (dimensionless)

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* Assistant Professor, Department of Theoretical and Applied Mechanics.
† Staff Engineer, Defense Operations Division.